Article
**ECORFAN Journal-Mexico**
**FINANCE**
December 2013 Vol.4 No.11 923-938

# Weather options valuations of fisheries sector in México

ALVA – Abraham*† & SIERRA- Guillermo

*Universidad se Guadalajara, Periferico Norte 799, Núcleo Universitario Los Belenes, 45100, Zapopan, Jalisco, México.*

Received April 01, 2013; Accepted October 29, 2013

The main objective of this paper develops a model of weather derivatives whose underlying physical variable is the temperature of the sea and has an application for coverage in the Mexican Pacific Fisheries Sector especially its relation to the natural phenomenon "El Niño". Historical information on the sea temperature is taken from different regions of the Mexican Pacific in order to propose a stochastic process describing the evolution of the temperature of the sea. As a first point is modeled the temperature, also taking into account that this is an underlying weather that cannot be traded, is used a market price of risk constant, which is an important parameter to calculate the prices of options contracts climatic into a derivatives market incomplete. We present the application of the model for the industry in some regions of the Mexican Pacific Fisheries Sector using the Monte Carlo simulation method. Also shows the specifications that should have some weather options contracts as well as numerical examples of prices for these contracts.

**Climate options, Black-Scholes equation, derived**

**Citation:** Alva A. & Sierra G. Weather options valuations of fisheries sector in México. ECORFAN Journal-Mexico 2013, 4-11: 923-938

*Correspondence to Author (email: aava840209duca@alumnos.cucea.udg.mx)
† Researcher contributing first author.

Article
ECORFAN Journal-Mexico
FINANCE
December 2013 Vol.4 No.11 923-938

## Introduction

The Weather Derivatives Market: The weather derivatives market is relatively young versus its counterpart of financial derivatives. The first trade operations in the weather derivatives market took place in the United States (US) in 1996 and 1997 (Jewson, 2005). The market was jump started during the El Niño winter of 1997–1998, which was one of the strongest such events on record. Many companies then decided to hedge their seasonal weather risk due to the risk of significant earnings decline (Alaton, Djehiche and Stillberger, 2002).

The first organized market where standard weather derivatives could be trade Chicago Mercantile Exchange was (CME). The CME offers futures and options contracts with monthly and seasonal periods based on temperature, rain, snowfall, humidity or hurricanes indices in 24 cities of US, six in Canada, 10 in Europe, two in Asia-Pacific y three cities in Australian. Weather products has grown from 2.2 USD billion in 2004 to 18 USD billion in 2007, with volume of nearly a million contracts traded, CME (2005) and (Myers, 2007). Moreover, the pricing model proposed for these contracts is the presented by (Alaton, Djehiche and Stillberger, 2002) which is taken as the main reference for this paper with the antecedent (Alva and Sierra, 2010), besides the importance of the article on different papers in weather derivatives on temperature indices.

The second paper mentioned has similar background, but does not get to specify a pricing weather option, whereas if it gets in this article. Some of the papers related are those presented by (Brody, Syroka and Zervos, 2002) (Jewson, 2004), (Richards, Manfredo and Sanders, 2004), (Benth and Saltyté-Benth, 2005 and 2007), (Zapranis and Alexandridis, 2008), (Benth, Härdle and Cabrera, 2011).

In Mexico, does not exist unfortunately still a weather derivatives market, although there are different papers on financial instruments with respect to weather and other natural phenomena that occur in Mexico, papers like those presented by (Díaz and Venegas, 2001), (Trujillo and Navarro, 2002), (Ibarra, 2003), (López, 2003 and 2006), (Fernández and Gregorio, 2005), (Baqueiro and Sinha, 2005).

The Effects of El Niño Southern Oscillation (ENSO)

El Niño Southern Oscillation or only El Niño is defined by prolonged warming in the sea surface temperature when compared with the average. The accepted definition is a warming of at least 0.5°C averaged over the period 1950-1979, for at least six consecutive months in the region known as “Niño 3” (4 ºN-4 ºS, 150 ºW-90 ºW) (Trenberth, 1997). El Niño is not periodic, usually occurs every three to seven years and lasts 12 to 18 months (McPhaden, 2002).

Signals the occurrence of El Niño are not limited to tropical regions of Pacific Ocean, but affect places as distant as North America or South Africa; (Ropelewsky y Halpert, 1989). In Mexico, El Niño has serious implications, in general we can say that the winter rains are intensified and are weakened summer. In the center and north of the country, increase cold fronts in winter, while the drought appears and decrease the number of hurricanes in the Atlantic, Caribbean Sea and Gulf of Mexico in summer; (Magaña, 1998).

But there are many more ways that El Niño affects Mexico and brings economic loss consequences within the country, such as in agriculture and fisheries.

ISSN-Print: 2007-1582- ISSN-On line: 2007-3682

The Fisheries and Analyzing the Impacts of El Niño in Mexico

Fishing is very important for Mexico, mainly because the country has 11,592.77 kilometers of coastline, 8475.06 correspond to the Pacific coast, and 3117.71 to the Gulf of Mexico and Caribbean Sea, including islands, the continental shelf is about 394,603 km2, being higher in the Gulf of Mexico; also has 12,500 km2 of coastal lagoons and matting and has 6,500 km2 of inland waters such as lakes, ponds, and rivers. By establishing the regime of 200 nautical mile exclusive economic zone in 1976, remain under national jurisdiction: 2,946,885 km2 marina region (Cienfuentes, Torres and Frías, 2003). This large-scale coastal, promotes fishing, which satisfies the domestic market and allows surplus for export.

The records of the National Commission of Aquaculture and Fisheries (CONAPESCA) (SAGARPA, 2008) indicate that the total volume of national fish production by live weight is 1,745,424 tons, representing a total amount of 16,884,000 pesos. Main top producing states are Sonora, Sinaloa, Baja California and Baja California Sur with 77% of total national fisheries and aquaculture production.

It's important to say that these states have an important capture of species such as sardines, shrimp, tuna, squid, tilapia and oyster, because they represent 82% of the total production, equivalent to 76% of the total value of domestic production. Furthermore, these species account for 43.7% of exports, with 59.2% of total value of domestic production; (SAGARPA, 2008).

Historically, El Niño 1997-98 has been the event has received more interest from several sectors in Mexican society.

Within fisheries, two of the largest fisheries in the Mexican Pacific (sardines and squid) had significant decreases in their levels of production, because in 1997 and 1998 had a decrease of 212 thousand tons, equivalent to about 16 million dollars.; (Magaña, 2004).

The sardine fishery of the Gulf of California, recorded significant socio-economic losses from El Niño 97-98, because in this activity, the potential for direct employment is about 3000, but the El Niño reduced this number until 50%; (Magaña, 2004).

The weather has had a huge impact on several financial activities, currently the list of businesses subject to weather risk is large and includes, for example, energy producers and consumers, supermarkets, entertainment and agricultural industries, and of course, fishing industries.

Thus, trade in weather derivatives for these companies has reduced their risk in the market in the presence of a "bad" weather. The weather derivatives are financial contracts with payouts that depend on the weather in some form. The underlying variables can be for example temperature, humidity, rain or snowfall.

In this work, we have the hypothesis that a weather derivative that used the temperature of the sea such as underlying variable permit us modeling a financial hedge against economic losses in the fishing industry of the Mexican Pacific caused by increases on the sea surface temperature.

The main objective is to propose a pricing model for weather options for the sector fisheries of the Mexican Pacific with payouts depending on sea surface temperature.

ISSN-Print: 2007-1582- ISSN-On line: 2007-3682

Especially, we expect that using historical data from sea temperature, we can suggest a stochastic process to model the evolution of the temperature as an underlying. As from suggested model, we expect to find a pricing model of weather options in which the sea surface temperature exceeds a certain threshold, in order to propose a hedge system. Addition, we apply the model proposed to an especially case, to the Fisheries Sector of the Mexican Pacific.

This work is divided into three main parts: 1. Introduction, here we present the definition of El Niño and its effects into the sector fisheries of the Mexican Pacific, a review of weather derivatives, and the relevance of the fisheries in Mexico. 2. We present the antecedents of the weather derivatives, the model for the sea surface temperature, and the pricing model of the derivative. 3. In the last part, we show the results and conclusions.

## The Model

Antecedents of the Model:

The first transaction in the weather derivatives market took place in the US in 1997. Since then, different models have been proposed for valuing of weather derivatives, which are usually structured as swaps, futures, and call and put options based on different underlying weather indices. Some commonly indices used are heating degree-days (HDDs) and the cooling degree-days (CDDs), which were originate from the US sector energy.

In winter, HDDs are used to measure the demand for heating, and are thus a measure of how cold it is (the colder it is, the more HDDs there are). The definition used in the weather market is that the number of HDDs on a particular day is defined as

$$HDD_i = \text{máx}(T_0 - T_i, 0) \quad (1)$$

Where $HDD_i$ is the number of *HDD*s for day $i$, $T_i$ is the average of the temperatura for day $i$, and $T_0$ is a baseline temperature.

An $H_n$ index of HDDs over a period of $n$ days is defined as the sum of HDDs over all days during that period, this index is usually defined as:

$$H_n = \sum_{i=1}^{n} HDD_i \quad (2)$$

The CDDs are used in summer to measure the demand for energy used for cooling, and are thus a measure of how hot it is (the hotter it is, the more CDDs there are). The number of CDDs on a particular day $I$ is defined as:

$$CDD_i = \text{máx}(T_i - T_0, 0) \quad (3)$$

Where $CDD_i$ is the number of CDDs for day $i$, $T_i$ is the average of the temperature for day $i$, y $T_0$ is a baseline temperature.

As for HDDs, a $C_n$ index of CDDs over a period of $n$ days is defined as the sum of the CDDs over all days during that period:

$$C_n = \sum_{i=1}^{n} CDD_i \quad (4)$$

We see that the number of HDDs or CDDs for a specific day is just the number of degrees that the temperature deviates from a temperature level. It has become industry standard in the US to set this reference level at 65° Fahrenheit (18°C). The reason is that if the temperature is below 18°C people tend to use more energy to heat their homes, whereas if the temperature is above 18°C people start turning their air conditioners on, for cooling.

The temperature $T_i$ for day $i$ given a specific weather station is defined as:

ISSN-Print: 2007-1582- ISSN-On line: 2007-3682

December 2013 Vol.4 No.11 923-938

$$T_i = \frac{T_i^{máx} + T_i^{mín}}{2} \quad (5)$$

Where $T_i^{max}$ y $T_i^{min}$ denote the maximal and minimal temperatures measured measured in day *i*. In this work we took the temperature on degree Celsius.

The buyer of a HDD call, for example, pays the seller a premium at the beginning of the contract. In return, if the number of HDDs for the contract period is greater than the predetermined strike level, the buyer will receive a payout. The size of the payout is determined by the strike and the tick size. The tick size is the amount of money that the holder of the call receives for each degree-day ($HDD_i$ or $CDD_i$) above the strike level for the period. Often the option has a cap on the maximum payout unlike, for example, traditional options on stocks; (Alaton, Djehiche and Stillberger, 2002).

Usually, a weather option con be formulated by specifying the following parameters: the contract type (call or put), the contract period, a underlying index (HDD or CDD), a official weather station from which the temperature data are obtained, the strike level, the tick size, the maximum payout (if there is any).

To find a formula for the payout of an option, let $K$ denote the strike level and $\alpha$ the tick size. Let the contract period consist of $n$ days. If the period of the contract consists of $n$ days and using the definition from the equation (2), we can write the payout of an uncapped HDD call as:

$$\chi = \alpha \text{máx}(H_n - K, 0) \quad (6)$$

The payouts for similar contracts like HDD puts and CDD calls or puts are defined in the same way.

Modelling the Sea Surface Temperature.

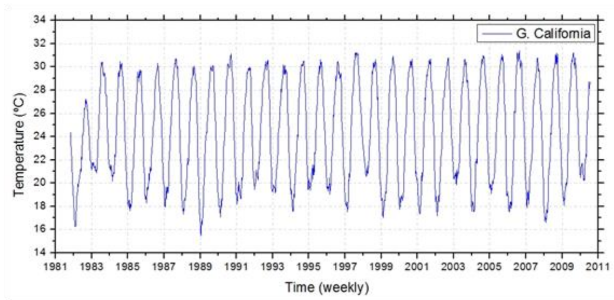
In this work, the main objective is to propose a pricing model for options depending of the sea surface temperature as the underlying variable. For this reason, is necessary propose a model that describes the temperature.

To help find a good model, we have a database with temperatures from November 1, 1981 to June 27, 2010 for different regions of the Mexican Pacific (Ensenada 117.5W-31.5N, Isla Cedros 115.5W-27.5N, Gulf of California 110.5W-26.5N, Cabo San Lucas 109.5W-22.5N, Puerto Vallarta 106.5W-20.5N, Acapulco 100.5W-16-5N and Gulf of Tehuantepec 94.5W-15.5N). The data series are weekly average temperatures obtained from (Reynolds, Rayner, Smith, Stokes and Wang, 2002). The source of data was obtained from the Climate Data Library from Columbia University; (IRI / LDEO, 2010). Graphic 1 shows the graph of the number of weekly average temperatures of the Gulf of California. In the figure it is clearly seen that there is a strong seasonal variation in the temperature, it appears that it should be possible to model the seasonal dependence with, for example, a sine-function. This function would have the form $\sin(\omega t + \varphi)$, where t denotes the time, measured in weeks. Since it is known that the period of the oscillations is one year (neglecting leap years) we have $\omega = 2\pi/365$. Because the yearly minimum and maximum mean temperatures do not usually occur at January 1 and July 1, respectively, a phase angle $\varphi$ must be introduced.

Moreover, a closer look at the data series reveals a positive trend in the data. It is weak but it does exist. The mean temperature actually increases each year. There can be many reasons for this. One is the fact that there may be a global warming trend all over the world.

ISSN-Print: 2007-1582- ISSN-On line: 2007-3682

Article
**ECORFAN Journal-Mexico**
**FINANCE**
December 2013 Vol.4 No.11 923-938




**Graphic 1**

We propose a deterministic model for the mean temperature $T_t^m$ at the time $t$, which would have the form:

$$T_t^m = A + Bt + C\sin(\omega t + \varphi) \qquad (7)$$

Where, the parameters A, B, C and $\varphi$ have to be chosen so that the curve fits the data well.

Using the equation (8), we estimate the numerical values of the constants in the equation (7) fitting to the temperature data using the method of least squares.

$$Y_t = a_1 + a_2 t + a_3\sin(\omega t) + a_4\cos(\omega t) \qquad (8)$$

This means finding the parameter vector $\xi = (a_1, a_2, a_3, a_4)$, that solves

$$\min_{\xi} \|\mathbf{Y} - \mathbf{X}\|^2 \qquad (9)$$

Where Y is the vector with elements in (7) and X is the data vector. The constants in the model (8) are then obtained as

$$A = a_1 \qquad (10)$$

$$B = a_2 \qquad (11)$$

$$C = \sqrt{a_3^2 + a_4^2} \qquad (12)$$

$$\varphi = \tan^{-1}\left(\frac{a_4}{a_3}\right) - \pi \qquad (13)$$

Inserting the numerical values into equation (7), we obtained the values that show in the table 1.

| Regions | *A* | *B* (X10$^{-9}$) | *C* | *φ* |
|---|---|---|---|---|
| Ensenada | 17.63 | 8.77 | 2.65 | -2.83 |
| Isla Cedros | 18.73 | 0.08 | 2.69 | 0.07 |
| G. California | 24.39 | 18.98 | 5.97 | -2.67 |
| Cabo San Lucas | 25.13 | 12.25 | 3.59 | 15.71 |
| Pto. Vallarta | 26.83 | 10.43 | 3.06 | -2.95 |
| Acapulco | 28.95 | 3.51 | 1.04 | -2.84 |
| G. Tehuantepec | 28.51 | 3.79 | 1.66 | -2.20 |

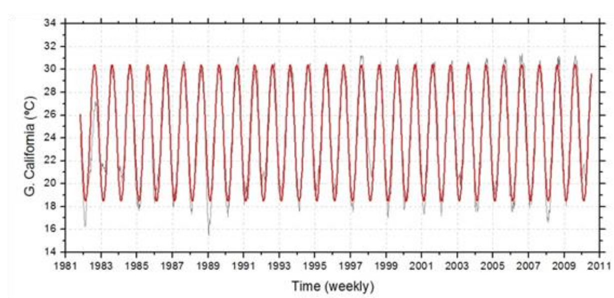
**Table 1**

As shown in Table 1 the amplitude of the sine function is different for each region, this might be due to temperature anomalies caused by El Niño.

Also, we can observe that the temperature decreases as it goes north, as we expected. From Table 1, we find that the function of Gulf of California $T_t^m$ average temperature is:

$$T_t^m = 24.39 + 18.98\text{X}\, 10^{-9}\, t + 5.97\sin(2\pi/365\, t - 2.67) \qquad (14)$$

The graph of function (14) with the data series of the sea temperature is shown in the graphic 2.

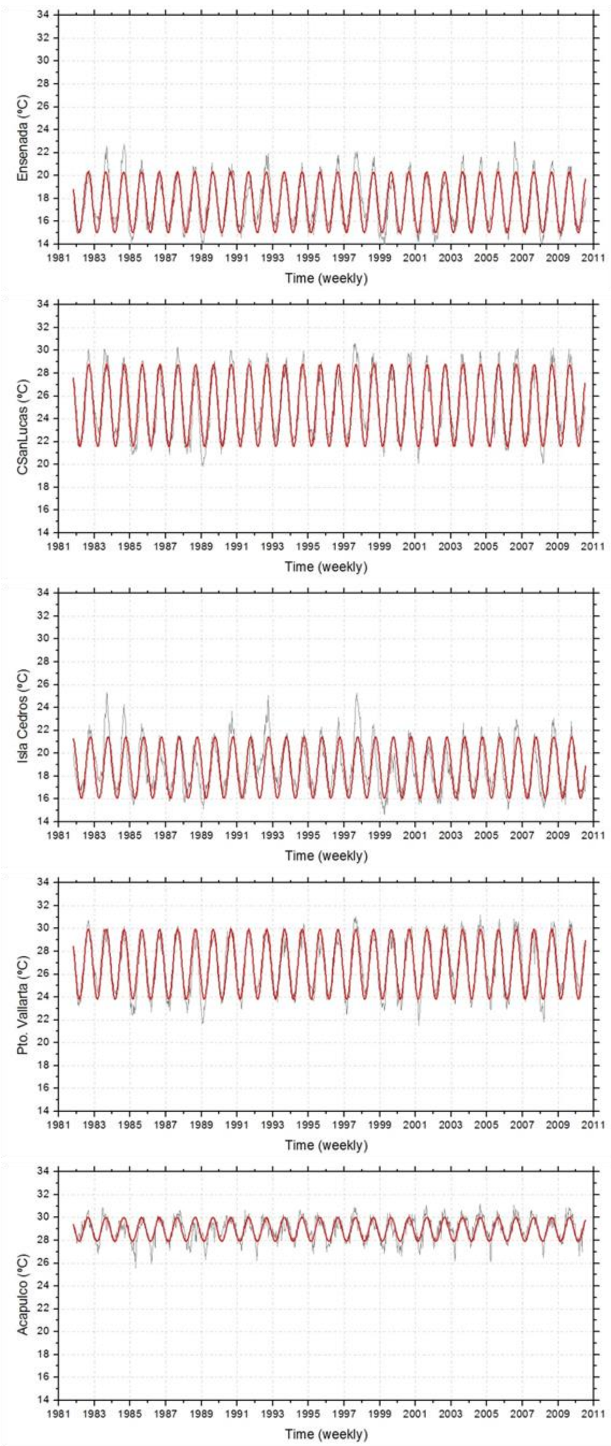


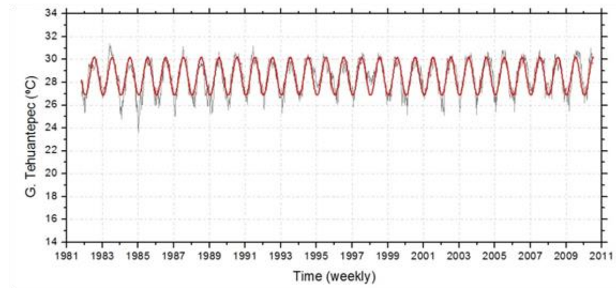

**Graphic 2**

ISSN-Print: 2007-1582- ISSN-On line: 2007-3682

Article
ECORFAN Journal-Mexico
FINANCE
December 2013 Vol.4 No.11 923-938

The graphs of the function (7) for different values from Table 1 together with the temperature data to the other regions from the study are shown in Graphic 3.

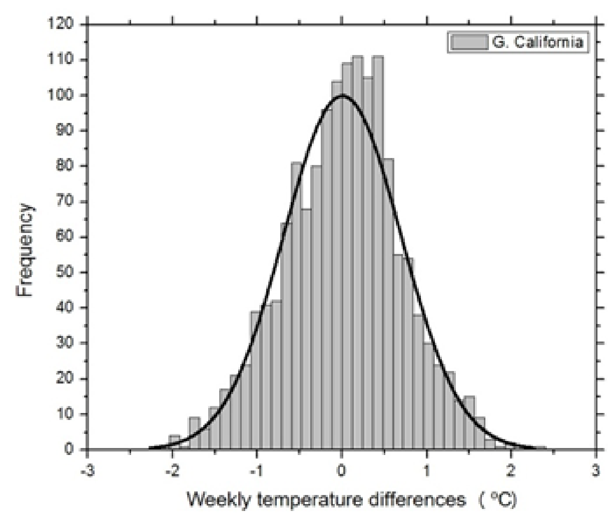



**Graphic 3**

Unfortunately, temperatures are not deterministic. Thus, to obtain a more realistic model we now have to add some sort of noise to the deterministic model (7).

We thought a standard Wiener process $(W_t, t \geq 0)$ is rigth. Indeed, this is reasonable not only with regard to the mathematical tractability of the model, but also because graphic 4 shows a good fit of the plotted weekly temperature differences with the corresponding normal distribution, though the probability of getting small differences in the weekly mean temperature will be slightly underestimated (Alaton, Djehiche and Stillberger, 2002).




**Graphic 4**

ISSN-Print: 2007-1582- ISSN-On line: 2007-3682

The quadratic variation $\sigma_t^2 \in R^+$ of the temperature varies across the different months of the year, but is nearly constant within each month. For example, during the summer and the winter the quadratic variation for the different regions is much higher than during the rest of the year. Therefore, the assumption is made that $\sigma_t$ is a piecewise constant function, with a constant value during each month. $\sigma_t$ is specified as:

$$\sigma_t = \begin{cases} \sigma_{1,} & during\ January, \\ \sigma_{2,} & during\ February, \\ \vdots & \\ \sigma_{12,} & during\ December, \end{cases}$$

Where $\{\sigma_t\}_{i=1}^{12}$ are positive constants. Thus, a driving noise process temperature would be $(\sigma_t W_t, t \geq 0)$; (Alaton, Djehiche and Stillberger, 2002).

For other side, it is also known that the temperature cannot, for example, rise day after day for a long time. This means that a model should not allow the temperature to deviate from its mean value for more than short periods of time.

In other words, the stochastic process describing the temperature should have a *mean-reverting* property.

Then, putting all the assumptions together, temperature is modelled by a stochastic process solution of the following stochastic differential equation:

$$dT_t = a(T_t^m - T_t)dt + \sigma_t dW_t \qquad (15)$$

Where a ∈ R determines the speed of the mean-reversion. The solution of such an equation is usually called an Ornstein–Uhlenbeck process.

The problema with equation (15) is that it is actually not reverting to $T_t^m$ in the long run; (Dornier and Querel, 2000). To obtain a process that really reverts to the mean (7) we have to add the term

$$\frac{dT_t^m}{dt} = B + \omega C\cos(\omega t + \varphi) \qquad (16)$$

To the drift term in (15). As the mean temperature $T_t^m$ is not constant this term will adjust the drift so that the solution of the stochastic differential equation has the long-run mean $T_t^m$; (Alaton, Djehiche and Stillberger, 2002).

Therefore, starting in $T_s = x$ we now get the following model for the temperature:

〖dT〗_t = {( 〖dT〗_t^m)/dt + a(T_t^m − T_t)}dt + σ_t 〖dW〗_t, t > s
(17)

Whose solution is

T_t = (x − T_s^m ) e^(−a(t − s) ) + T_t^m + ∫_s^t▒ 〖e^(−a(t − τ) ) σ_τ 〖dW〗_τ 〗 (18)

Where:

$$T_t^m = A + Bt + C\sin(\omega t + \varphi)$$

According by Alaton, Djehiche and Stillberger, 2002, we drive two estimators of $\sigma$ from data collected for each month. Given a specific month $\mu$ of $N_\mu$ weeks, denote the outcomes of the observed temperatures during the month $\mu$ by $T_j$, $j = 1,\ldots,N_\mu$. The first estimator is based on the quadratic variation of $T_t$; (Basawa and Prasaka Rao, 1980) as:

$$\sigma_\mu^2 = \frac{1}{N_\mu}\sum_{j=0}^{N_\mu - 1}\left(T_{j+1} - T_j\right)^2 \qquad (19)$$

The second estimator is derived by discretizing (17) and thinking of the discretized equation as a regression equation. Thus, the second estimator of $\sigma_\mu$; Brockwell and Davis (1990) during a given month $\mu$ have the following form:

$$\sigma_\mu^2 = \frac{1}{N_\mu - 2} \sum_{j=1}^{N_\mu} \left(\tilde{T}_j - \hat{a}T_{j-1}^m - (1 - \hat{a})T_{j-1}\right)^2 \quad (20)$$

Where

$$\tilde{T}_j \equiv T_j - \left(T_j^m - T_{j-1}^m\right)$$

To find the estimate of $\sigma_\mu$ in Eq. (20), one needs to find an estimator of $a$. Therefore, it is appropriate to estimate the mean-reversion parameter $a$ using the martingale estimation functions method suggested bye (Bibby and Sørensen, 1995). Based on observations collected over $n$ weeks, an efficient estimator $\hat{a}_n$ of $a$, is given as; (Alaton, Djehiche and Stillberger, 2002):

$$\hat{a}_n = -\ln\left(\frac{\sum_{i=1}^n Y_i\{T_i - T_i^m\}}{\sum_{i=1}^n Y_{i-1}\{T_{i-1} - T_{i-1}^m\}}\right) \quad (21)$$

Where

$$Y_{i-1} \equiv \frac{T_{i-1}^m - T_{i-1}}{\sigma_{i-1}^2}, \quad i = 1,2,\ldots,n \quad (22)$$

Using the data of the temperatures into the equations (19) and (20) for the regions analyzed, we obtained the $\sigma$ that are listed in Table 2. As expected, the $\sigma$'s are different for each region, this can probably be attributed to El Niño, because in each region this phenomenon affects in different time of year.

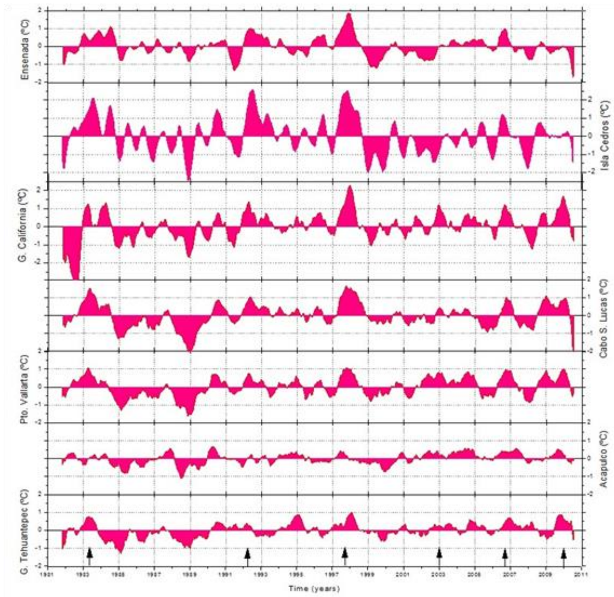
| Mes | ENS | IC | GC | CSL | PV | AC | GT |
|---|---|---|---|---|---|---|---|
| Enero | 0.29 | 0.34 | 0.46 | 0.44 | 0.45 | 0.29 | 0.72 |
| Febrero | 0.35 | 0.38 | 0.53 | 0.37 | 0.41 | 0.32 | 0.74 |
| Marzo | 0.49 | 0.53 | 0.67 | 0.51 | 0.53 | 0.48 | 0.67 |
| Abril | 0.52 | 0.50 | 0.68 | 0.48 | 0.53 | 0.56 | 0.54 |
| Mayo | 0.43 | 0.47 | 0.71 | 0.56 | 0.65 | 0.58 | 0.54 |
| Junio | 0.44 | 0.55 | 0.66 | 0.74 | 0.70 | 0.68 | 0.49 |
| Julio | 0.49 | 0.63 | 0.48 | 0.77 | 0.59 | 0.47 | 0.45 |
| Agosto | 0.44 | 0.53 | 0.43 | 0.51 | 0.57 | 0.46 | 0.37 |
| Septiembre | 0.50 | 0.73 | 0.49 | 0.59 | 0.52 | 0.44 | 0.42 |
| Octubre | 0.51 | 0.53 | 0.69 | 0.50 | 0.50 | 0.42 | 0.67 |
| Noviembre | 0.52 | 0.48 | 0.84 | 0.55 | 0.49 | 0.37 | 0.67 |
| Diciembre | 0.45 | 0.49 | 0.68 | 0.55 | 0.51 | 0.37 | 0.60 |

**Table 2**

Using the mean values of σ from the Table 2, we obtained the estimates of the mean reversion parameter for the different regions analyzed. These parameters are listed in the Table 3. From Table 3, we can observe that the speed of mean reversion for each region is different, this because (again probably some) to El Niño, the mean reversion parameter turned out to be smaller than in regions where El Niño does not affect in the same way (see Graphic 5).

| Region | Parameter $a$ |
|---|---|
| Ensenada | 0.103 |
| Isla Cedros | 0.070 |
| G. California | 0.075 |
| Cabo San Lucas | 0.116 |
| Pto. Vallarta | 0.143 |
| Acapulco | 0.233 |
| G. Tehuantepec | 0.278 |

**Table 3**




**Graphic 5**

When the signal of the temperature have a small value of mean reversion that means it take "more time" to return to its equilibrium level. In this case, the regions that are most affected by El Niño have more noise in the temperature signal, as shown in Figure 5, resulting a small value in its mean reversion parameter.

Now, having estimated all the unknown parameters in the temperature model (17) – (19), we are able to simulate trajectories of the Ornstein–Uhlenbeck (OU) process using Monte Carlo simulation. To do the simulation, we need to find from (17) an equation discretized. Thus, we solves the eq (18) between s and t, with t > s; (Dagpunar, 2007) so:

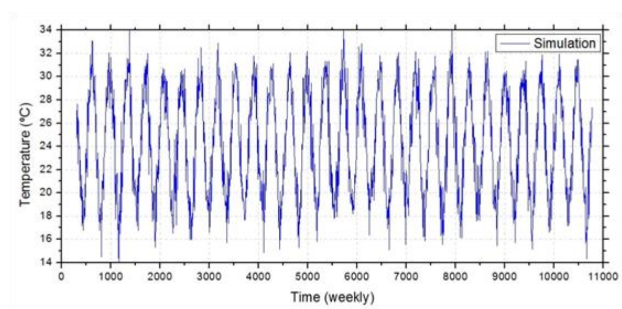
$$T\_t = (T\_s - T\_s\hat{}m)\, e\hat{}(-a(t-s)) + T\_t\hat{}m + \sigma\_\mu \sqrt{}((1 - e\hat{}(-2a(t-s)))/2a)\, W\_((s,t)) \qquad (23)$$

Where $\{W_{(s,t)}\}$ are independent standard normally distributed random variable for discrete intervals $\{(s,t)\}$.

To do the simulation of the OU process on the interval $\Delta t$, we obtained:

$$T\_(t+1) = (T\_t - T\_t\hat{}m)\, e\hat{}(-a\Delta t) + T\_(t+1)\hat{}m + \sigma\_\mu \sqrt{}((1 - e\hat{}(-2a\Delta t))/2a)\, \epsilon\_t \qquad (24)$$

Where $\epsilon_t$ is a number derived from a distribution $N(0,1)$, which were generated from Ziggurat Method. Thus, using (24) is possible simulated one trajectory of the temperature during the following years for the region of the Gulf of California. Comparing this simulation with the real temperatures plotted earlier in Figure 6, it is concluded that, at least visually, the temperature model (17) – (19) has the same properties as the observed temperature. However, we can note that the signal present a little more noise than the original signal of the sea surface temperature, this can probably be by the sigma parameter, because the estimation of the average of sigma is most higher than the original signal.




**Graphic 6**

Weather Derivatives Valuation.

Weather derivatives market is a classical example of incomplete market, particularly in México. The Mexican market of derivatives (Mercado Mexicano de Derivados MexDer ) began operations in December 1998.

We said that is an incomplete market because the temperature is not a negotiable asset.

For that reason we should consider the risk price in order to obtain the correct price also we assume constant price $\lambda$.

According to (Alaton, Djehiche y Stillberger, 2002), they assume an constant risk free interest rate and the contract paid a specific value for each degrees Celsius under a martingale measure **Q**, an with $\lambda$, the process for the temperature $T_t$, follow the expression

$$dT_t = \left\{\frac{dT_t^m}{dt} + a(T_t^m - T_t) - \lambda\sigma_t\right\}dt + \sigma_t dV_t \quad (25)$$

Where $(V_t, t \geq 0)$ is a **Q**-Wiener process, the valuation of one derivative contract is mentioned like a expected discount value under martingale measure **Q**. According (Alaton, Djehiche y Stillberger, 2002) the expected value and the variance of $T_t$ under are;

$$\mathrm{E}^{\mathbf{Q}}[T_t|\mathcal{F}_s] = (T_s - T_s^m)e^{-a(t-s)} + T_t^m - \frac{\lambda\sigma_i}{a}\left(1 - e^{-a(t-s)}\right) \quad (26)$$

$$\mathrm{Var}[T_t|\mathcal{F}_s] = \frac{\sigma_i^2}{2a}\left(1 - e^{-2a(t-s)}\right) \quad (27)$$

Then, for the simulation of paths under risk neutral measure **Q**, we must include $\lambda$ in equations (23) y (24) and using equations (26) y (27) in order to simulate an Ornstein-Uhlenbeck (OU) process $t > s$ we obtain:

$$T_t = (T_s - T_s^m)e^{-a(t-s)} + T_t^m - \frac{\lambda\sigma_i}{a}\left(1 - e^{-a(t-s)}\right) + \sigma_\mu\sqrt{\frac{1-e^{-2a(t-s)}}{2a}}W_{(s,t)} \quad (28)$$

Where $\{W_{(s,t)}\}$ are independent random variables for discontinuous intervals on $\{(s,t)\}$. And for simulate a process (OU) in each interval $\Delta t$, we obtain:

$$T_{t+1} = (T_t - T_t^m)e^{-a\Delta t} + T_{t+1}^m - \frac{\lambda\sigma_i}{a}(1 - e^{-a\Delta t}) + \sigma_\mu\sqrt{\frac{1-e^{-2a\Delta t}}{2a}}\epsilon_t \quad (29)$$

Where $\epsilon_t$ are Gaussian random numbers $N(0,1)$, generated from Ziggurat (Marsaglia y Tsang 2000) method.

Application of the Model to Fisheries Sector in México.

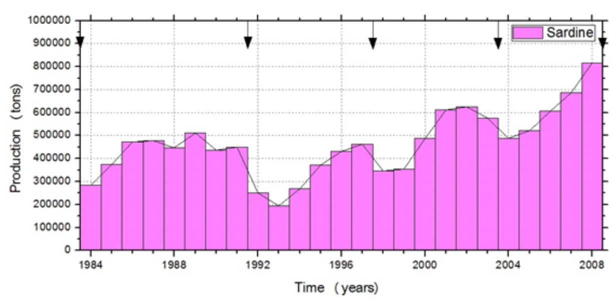
This paper is specially interested in the sardine fishing in Golf de California region because of importance of volume of fishing production and the anomalies of sea temperature of the phenomenon called "El niño"

First we assume $\lambda$ as a constant parameter because there are not market operations in Mexico and we cannot compare real price of contracts. Besides we need to a level of reference of temperature, an equivalent of exercise price for the temperature and one nominal value $\alpha$.

The reference of temperature for weather derivatives on United States and some european countries for environment temperature is 18 °C. In this case we propose to consider the sea temperature and we made a hedging for consequences of the natural effects of phenomena "El niño".

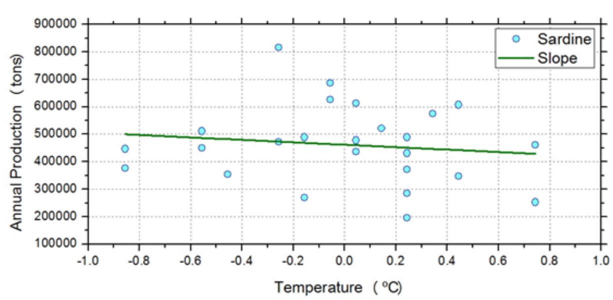
For this work we propose that the reference level for a call option *heating degree-week* (HDW) for sardine fishing in Golf of California be 20 °C because the sardine would prefer to live on the interval between 17 and a 20 °. If the temperature grow up above 3 °C the percentage of mortality could increase 40 % (Hernández y Barón 2009).

Other value that it should be estimated in an incomplete market is the nominal value $\alpha$ that correspond to the amount that the buyer o seller received for each degree Celsius every week on *heating degree-week* or *cooling degree-week* ($HDW_i$ o $CDW_i$ ) that received over exercise price (K) during the life of the contract. In order to obtain a nominal value $\alpha$, we propose to analyze the change in the weight production o fish capture (see Graphic 7) versus sea temperature on Mexican sea for the period 1984-2008 using a linear regression analysis between sea temperature and how much the production have fallen in sea with the change of temperature.




**Graphic 7**

The graphic 8 shown the results of the lineal regression analysis and we considered the beda period. (OEIDRUS Sonora 2005). The table 5 describe the slope and the intercept for the case of sardine.




**Graphic 8**

| Production | Slope (*tonnes*/°C) | Intercept *b* (*tonnes*) |
|---|---|---|
| Y = *a*X + *b* | -44116.86 | 462208.30 |

**Table 4**

On Golf de California region the lowest sea temperatures are from December to March (in the interval from 17 to 20 °C). However the highest volumes are registered on March. On that month because of the "EL Niño" phenomenal the production is reduce significantly respect the year before production.

We can see on table 4 that if the sea temperature increased in one degrees celcius the sardine production would be reduce on 44,000 tonns (comparing march 2010 and march 2011). Then we could propose a contract for a four weeks period corresponding to march 2011 a nominal value of 11,000 *tonns*/HDW, according (SAGARPA, 2008) the approximate value should be of MX$6,000,000 *pesos*/*HDW* ( see Graphic 5).

Specifications of the contracts

We know that the equivalent of the exercise price K is related with the period of the contract, we propose that a period of four weeks corresponding to the march 2011 the exercise level is 4 *HDW* because we suppose that each week the temperature increase 1 °C respect the reference temperature $T_0$ as a result as one anomaly (see Graphic 5). The specifications contract of HDW call options is listed don table 6.

In the model We propose a value of $\lambda$ = 0 y one value of $K$ = 4 *HDW*. We could repeat the process and obtain the exercise level for January and February. The results are show in Table 5.

| Parameters | Option I | Option II | Option III |
|---|---|---|---|
| Region | G. de California | G. de California | G. de California |
| Index | HDW | HDW | HDW |
| Type | Call | Call | Call |
| Period | January 2011 | February 2011 | March 2011 |
| Free Risk rate $r$ | 5% | 5% | 5% |
| Ref. level $T_0$ | 20 °C | 20 °C | 20 °C |
| Exercise $K$ | 5.28 *HDW* | 7.35 *HDW* | 4.00 *HDW* |
| Notional $\alpha$ | 6,000,000 MX$/HDW | 6,000,000 MX$/HDW | 6,000,000 MX$/HDW |

**Table 5**

We should mention that the information is weekly for that reason the value of the temperature options is minus that the other *HDD* y *CDD.*

## Results and conclusions

For this problem do not exit an explicit formula for the weather option valuation then we appeal another technical solution, Montecarlo simulation. The method essentially repeat one process and the end estimate the expected value.

The Gaussian random numbers are generated with Ziggurat algorithmic using the MATLAB version 7.6. We built 100,000 paths and after the estimate the premium of the different options. (See table 6 for different $\lambda$. Values). There are only some cases of the valuation of weather derivatives probably of similar characteristics.

| $\lambda$ *Value* | Option I | Option II | Option III |
|---|---|---|---|
| $\lambda = 0.00$ | 3.3 | 3.3 | 3.3 |
| $\lambda = 0.01$ | 3.8 | 3.8 | 3.8 |
| $\lambda = 0.02$ | 4.4 | 4.4 | 4.4 |
| $\lambda = 0.025$ | 4.7 | 4.8 | 4.7 |
| $\lambda = 0.05$ | 6.4 | 6.6 | 6.5 |
| $\lambda = 0.075$ | 8.4 | 8.9 | 8.7 |
| $\lambda = 0.10$ | 10.8 | 11.6 | 11.2 |

**Table 6**

The weather option buyer usually paid a premium to the seller between 10% and 20 % of the notional value of the contract.

We can observe Table 6 value of $\lambda$ since 0 to 0.025 and premium are between 14 y 20% of the contract notional (MX$ 24,000,000 pesos).

The weather option premium could vary significantly depend of risk profile.

On another hand for if the option premium (for $\lambda > 0$) is higher than 14% of the notional contract obeys to the sensibility of the sardine production with the changes of sea temperature.

## Conclusions

In the literature about the weather derivatives practically we have not found other works that consider derivatives of sea temperature. This paper proposes the characteristics, specifications and the method of valuation of a contract of weather option in order to hedging the possible sardine production loss as consequence of natural phenomena “El Niño”.

The model considers the sea temperature as underlying and following a combination of stochastic process, a drift and the cycles of temperature on the year. The exercise temperature is threshold of temperature when the sardine begins to die or migrate to other region. The weather option premium fluctuates from 10 to 20% of the notional contract. The valuation is very sensible to the volatility of the temperature, the market risk and the period of the contract.

However we should recognize that in Mexico there not market for weather derivatives although we think that because of the necessity of hedging for natural disaster in short term some insurance company could operate weather derivatives, specially fisheries and farm sector.

## References

Alaton, P., Djehiche, B. y Stillberger, D., (2002). On modeling and pricing weather derivatives, *Applied Mathematical Finance 9(1)*: 1-20.

Alva A, Sierra G,.(2010) Opciones climáticas para el sector pesquero del pacific" Panaroma Económico, Numero 11. Jul-Dic 2010

Baqueiro C., E. y Sinha, T., (2005). *Seguros de lluvia mediante el uso de derivados*, Tesis.

Basawa, I. V. y Prasaka Rao, B. L. S., (1980). *Statistical Inference for Stochastic Process*, Academic Press.

Benth, F. E., Härdle, W. K. y López C., B., (2011). Pricing of Asian temperature risk in Statistical, Tools for Finance and Insurance, *2aEd, Springer Verlag Heilderberg*.

Benth, F. E. y Šaltytė-Benth, J., (2005). Stochastic Modelling of Temperature Variations with a View Towards Weather Derivatives, *Applied Mathematical Finance 12(1)*: 53-85.

Benth, F. E. y Šaltytė-Benth, J., (2007). The volatility of temperature and pricing of weather derivatives, *Quantitative Finance 7(5)*: 553-561.

Bibby, B.M. y Sørensen, M., (1995). Martingale estimation functions for discretely observed diffusion processes, *Bernoulli 1(1/2)*: 17-39.

Bravo C., J.L., Azpra R., E., Zarraluqui S., V., Gay G., C. y Estrada P., F. (2010). Significance tests for the relationship between "El Niño" phenomenon and precipitation in Mexico, *Geofísica Internacional 49(4)*: 245-261.

Brockwell, P. J. y Davis, R. A., (1990). *Time Series: Theory and Methods*, 2a ed. Springer.

Brody, D., Syroka, J., y Zervos, M., (2002), Dynamical pricing of weather derivatives, *Quantitative Finance (3)*: 189-198.

Cienfuentes L., J. L., Torres G., P. y Frías M., M., (2003). *El océano y sus recursos, IX. La pesca (81)*, 5a ed. (La ciencia para todos) Fondo de Cultura Económica, México, D.F.

CME, (2005). An introduction to cme weather products, http://www.levow.com/SGdownload/-Commodities/CME Weather products.pdf, *CME Alternative Investment Products*.

Dagpunar, J. S., (2007). *Simulation and Monte Carlo with applications in Finance and MCMC*, John Wiley & Sons, London.

Díaz T., J. y Venegas M., F., (2001). Política Agrícola y Contratos de Futuros: Un Modelo de Arbitraje, *Momento Económico (115)*: 2-21.

Fernández D., J. J. y Gregorio D., M. M., (2005). Valuación Actuarial de Bonos Catastróficos para Desastres Naturales en México, *El Trimestre Económico 72(288)*: 877-912.

Hernández, M. y Barón, B., (2009). Sardina, especie indicadora de variabilidad climática, *Gaceta CICESE 136, Todos@CICESE, Sección Ciencia y Tecnología*. http://gaceta.cicese.mx/.

Ibarra P., H., (2003). *Administración de riesgos naturales en México: un caso práctico en el diseño de un derivado climático para la agricultura*, Tesis.

IRI/LDEO, (2010), Climate Data Library, IGOSS nmc Reyn SmithOIv2 weekly sst http://iridl.ldeo.columbia.edu/SOURCES/.IGOSS/.nmc/.Reyn SmithOIv2/.weekly/.sst/

Jewson, S., (2004). Introduction to Weather Derivative Pricing, *The Journal of Alternative Investments*: 57-64.

Jewson, S. y Brix, A., (2005). *Weather Derivative Valuation: The Meteorological, Statistical, Financial and Mathematical Foundations*, Cambridge University Press.

López C., B., (2003). *Valuación de Bonos Catastróficos para terremotos en México*, Tesis.

López C., B., (2006). *Pricing Catastrophic Bonds for Earthquakes in Mexico*, Masther Tesis.

Magaña, V., Pérez, J.L. y Conde, C., (1998). El fenómeno de El Niño y la Oscilación del sur y sus impactos en México, *Revista Ciencias 51*: 14-18.

Magaña R., V., (ed) (2004). *Los Impactos de El Niño en México*, Centro de Ciencias de la Atmósfera, UNAM, Secretaria de Gobernación, 229 p.

Marsaglia, G. y Tsang, W. W. (2000). The Ziggurat Method for Generating Random Variables, *Journal of Statistical Software, 5(8)*.

McPhaden, M., (2001). El Ni~no and La Ni~na: causes and global consequences, En: M. Cracken y J. Perry (ed), *Encyclopedia of global environmental change 1*, John Wiley & Sons, New York: 929-948.

Moler, C. B. (2004). *Numerical Computing with MATLAB*, SIAM: 336pp, http://www.mathworks.com/moler.

Myers, R., (2007). What Every CFO Needs to Know Now About Weather Risk Management, http://www.cmegroup.com/trading/weather/, *CME Group*.

OEIDRUS, Sonora (2005), Oficina Estatal de Información para el Desarrollo Rural Sustentable del Estado de Sonora, *Sonora. Sector Pesquero (Aspectos Físicos y Disponibilidad de Recursos)*. http://www.oeidrus-sonora.gob.mx/index 22.php

Reyes, S. y Troncoso, R. (2004). Modulación multidecenal de la lluvia en el Noroeste de Baja California, *Rev. Cienc. Mar. 30*: 99-108.

Reynolds, R. W., Rayner, N.A., Smith, T. M., Stokes, D.C. y Wang, W., (2002). An Improved In Situ and Satellite SST Analysis for Climate, *J. Climate 15*: 1609-1625.

Richards, T. J., Manfredo, M. R. y Sanders, D. R., (2004). PricingWeather Derivatives, *Amer. J. Agr. Econ. 86(4)*: 1005-1017.

Ropelewsky, C.F. y Halpert, M.S., (1989). Precipitation patterns associated with the high index phaase of the Southern Oscillation, *J. Climate 2*: 268-284.

SAGARPA, (1984-2008). Secretaría de Agricultura, Ganadería, Desarrollo Rural, Pesca y Alimentación, CONAPESCA, Anuarios Estadísticos de Acuacultura y Pesca de 1984 a 2008. http://www.conapesca.sagarpa.gob.mx/wb/cona/cona_anuario_estadistico_de_pesca.

SEMARNAT, (2009). Secretaria de Medio Ambiente y Recursos Naturales, *Compendio de Estadística Ambiental*, http://www.semarnat.gob.mx/.

Trenberth, K.E., (1997). The deifnition of El Niño, *Bull. Am. Meteorol. Soc. 78*: 2771-2777.

Trujillo P., M. L. y Navarro T., R. C., (2002). *Derivados Climáticos*, Tesina.

ISSN-Print: 2007-1582- ISSN-On line: 2007-3682

Zapranis, A. y Alexandridis, A., (2008). Modelling the Temperature Time-dependent Speed of Mean Reversion in the Context of Weather Derivatives Pricing, *Applied Mathematical Finance 15(4)*: 355-386.

ISSN-Print: 2007-1582- ISSN-On line: 2007-3682